Detailed description of the determination of winning bids for the fourth auction stage

June 5, 2018

1 Overview

Winning bids for the fourth auction stage are determined by maximising the sum of the differences between the maximum price reduction available for a bidder taking on the obligation to provide coverage of a particular address group and the amount of the selected bid (objective function) subject to selecting at most one bid for each address goup, and not selecting bids from a bidder with a total value that exceeds the bidder's provisional licence price (constraints).

The winner determination process can be implemented as a simple linear programme (though some of the tie breaking criteria will involve quadratic programming).

2 Notation

Let T be the set of lots (address groups) available, and ρ_k denote the maximum price reduction for lot $k \in T$ (which is the maximum amount that bidders can specify in their bids).

Let S be the set of bidders taking part in the fourth auction stage, B^j the set of bids submitted by bidder $j \in S$, and $\beta_k^j \in B^j$ be the bid from bidder $j \in S$ for lot $k \in T$ (i.e. the reduction in the licence price for which the bidder offers to take on the coverage obligation for the address group k). Let λ^j be the provisional licence price for bidder j.

Define a binary variable ϕ_k^j which takes value 1 if β_k^j is selected, and zero otherwise, so that $\Phi = \left\{ \phi_k^j \right\} \quad \forall j \in S, \forall k \in T$ denotes as set of selected bids.

Our objective function can then be written as :

$$V(\varPhi) = \sum_{j \in S} \sum_{\beta_k^j \in B^j} \phi_k^j \left(\rho_k - \beta_k^j \right)$$

and the constraints are:

$$\sum_{j \in S} \phi_k^j \leq 1 \quad \forall k \in T \tag{2.1}$$

$$\sum_{\beta_k^j \in B^j} \phi_k^j \beta_k^j \leq \lambda^j \quad \forall j \in S \tag{2.2}$$

3 Selection of winning bids

3.1 Value-maximising combinations of bids

Let $\omega = \{\Phi^*\}$ be the set of sets of selected bids that maximise the objective function subject to the constraints in (2.1) and (2.2). If $|\omega| = 1$, i.e. the solution to this maximisation problem yields a unique set of bids, this will be selected as the set of winning bids for the fourth auction stage. If multiple combinations of bids produce the same highest value of the objective function, ties will be broken through a hierarchy of tie breaking criteria.

3.2 Tie-breaking criteria

First, we select the set(s) of bids in $\omega^\star \subseteq \omega$ that maximise the sum of maximum price reductions of lots that are assigned, i.e.

$$\omega^\star = \left\{ \mathit{\Phi} : \arg\max_{\mathit{\Phi}} \sum_{j \in S} \sum_{\beta_k^j \in B^j} \phi_k^j \rho_k, \; s.t. \, \mathit{\Phi} \in \omega \right\}$$

If there is only one element in $\omega^\star$, this will be selected as the set of winning bids for the fourth auction stage. Otherwise, we identify the set of combinations $\omega^{\star\star} \subseteq \omega^\star$ that maximise the number of lots assigned, i.e.

$$\omega^{\star\star} = \left\{ \mathit{\Phi} : \arg\max_{\mathit{\Phi}} \sum_{j \in S} \sum_{\beta_k^j \in B^j} \phi_k^j, \; s.t. \, \mathit{\Phi} \in \omega^\star \right\}$$

If there is only one element in $\omega^{\star\star}$, this will be selected as the set of winning bids for the fourth auction stage. Otherwise, we identify the set of combinations $\omega^{\star\star\star} \subseteq \omega^{\star\star}$ that minimse sum of the squares of the bidders' total reductions across all bidders, i.e.

$$\omega^{\star\star\star} = \left\{ \mathit{\Phi} : \arg\min_{\mathit{\Phi}} \sum_{j \in S} \left(\sum_{\beta_k^j \in B^j} \phi_k^j \beta_k^j \right)^2, \; s.t. \, \mathit{\Phi} \in \omega^{\star\star} \right\}$$

If there is only one element in $\omega^{\star\star\star}$, this will be selected as the set of winning bids for the fourth auction stage. Otherwise, we select at random one of the combinations in $\omega^{\star\star\star}$ as the set of winning bids for the fourth auction stage.